Sets

item	i	i	€{1,2,….,I}
distribution centre	d	d	€{1,2,…,d}
customer zone	c	c	€{1,2,..,C}
plant	p	p	€{1,2,..,P}
time period (month)	t	t	€{1,2,..,T}

List of symbols and parameters

Dem_{ict}	The demand for item i at customer zone c in time period t.
$hcapa^{e}_{idt}$	The available inventory capacity for item i at d/c (distribution center) d in time period t
$capa^{PA}_{p\,t}$	The total available production capacity for plant p in time period t.
$reqcapa_{ipt}$	The requested(desired) production quantity for item i at plant p f in time period t
rt^{p}_{ip}	The inventory capacity utilization rate of product I in time period t.
$mcapa^{f}_{pt}$	available production capacity for plant p in time period t
$capa^{i}_{pt}$	The total The production capacity utilization rate per unit of item i in plant p
rt^{i}_{ip}	The inventory capacity utilization rate per unit of item i in plant p
$capa^{f}_{pt}$	The total available inventory capacity for plant p in time time period t
$Cost^{1}_{ict}$	The unit lateness cost for item i at customer zone c in time period t
$Cost^{2}_{idt}$	The unit earliness cost for item i at d/c d in time period t
$Cost^{3}_{ipdt}$	The unit transportation cost for item i from plant p to d/c d in time period t
$Cost^{4}_{idct}$	The unit transportation cost for item i from d/c d to customer zone c in time period t
$Cost^{5}_{ip\,t}$	The unit production cost for item i at plant p in time period t
$Cost^{6}_{ip\,t}$	The unit inventory holding cost for item i in plant p in time period t
$Cost^{7}_{ipt}$	The unit unfulfilled penalty cost against the requested quantity from the DA for item i at plant p in time period t.(set to a very big number)

3.1.3 Decision Variables

$late_{ict}$	The lateness quantity for item i at customer zone c in period t.
$earli_{idt}$	The earliness quantity for item i at d/c d in time period t.
$transpd_{ipdt}$	The transportation quantity for item i from plant p to d/c d in time period t
$transdc_{idct}$	The transportation quantity for item i from d/c d to customer zone c in time period t
$fcapa_{ipt}$	The required(desired) production quantity for item i at plant p in time period t
y_{ipt}	The production quantity for item i at plant p in time period t

inv_{ipt} The inventory quantity for item i at plant p in time period t

q_{ipt} The unfulfilled quantity against the requested quantity for item i at plant p in time period t

Product Symbols

P1 : MMB : Mono-block pumps

P2 : CP : centrifugal pumps

P3 : JET : Jet pumps

P4 : SMB : Submersible Mono-block pumps

P5 : HP : Hand pumps

P6 : Motors : Electric motors

P7 : SP :" Submersible pumps

P8 : PP ; Piston pumps

3.2.2 Other Symbols

F1 : First factory

F2: Second Factory

DC1 : First Distribution centre

DC2 : Second distribution centre

C1, C2. C3, C4 : Customer zones.

Formulation of the centralized production-distribution planning model

$$\text{Min} \sum_{i,c,t} Cost^{1}_{i,c,t}\, late_{ict} + \sum_{i,d,t} Cost^{2}_{idt}\, earli_{idt} + \sum_{ipdt} Cost^{3}_{ipdt}\, transpd_{ipdt} + \sum Cost^{4}_{idct}\, transdc_{idct} + \sum_{i,p,t}\left(Cost^{5}_{ipt}\, y_{ipt} + Cost^{6}_{ipt} inv_{ipt} + Cost^{7}_{ipt} q_{ipt}\right) \quad (14)$$

Subject to

$$\sum_{d\in D(c)} transdc_{idct} = Dem_{ict} - late_{ict} \quad \forall\, i,c,t \quad (15)$$

$$eali_{\,i,d,t-1} + \sum_{p\in P(d)} transpd_{ipdt} = earli_{idt} + \sum_{c\in C(d)} transdc_{idct} \forall i,d,t \quad (16)$$

$$earli_{idt} \leq hcapa^{e}_{idt} \quad \forall\, i,d,t \quad (17)$$

$$\sum_{m \in F(f)} transpdd_{ipdt} = fcapa_{ipt} \qquad \forall i, p, t \tag{18}$$

$$reqcapa_{ipt} = fcapa_{ipt} \qquad \forall i, p, t \tag{19}$$

$$inv_{i,p,t-,1} + y_{ipt} = reqcapa_{ipt} + inv_{ipt} - q_{ipt} \qquad \forall i, p, t \tag{20}$$

$$\sum_i util_{ip}^{p} y_{ipt} \leq mcapa_{pt}^{f} \forall p, t \tag{21}$$

$$\sum_i util_{ip}^{i} inv_{ipt} \leq icapa_{pt}^{i} \forall p, t \tag{22}$$

$$late_{ict} \geq 0, earli_{i,d,t} \geq 0, transpd_{ipdt} \geq 0, transdc_{idct} \geq 0, \tag{23}$$

$$reqcapa_{ipt} \geq 0, y_{ipt} \geq 0, inv_{ipt} \geq 0, q_{ipt} \geq 0 \qquad \forall i, p, d, c, t$$