Both families of constraints (1.8) and (1.12) have a cardinality growing exponentially with n. This means that it is practically impossible to solve directly the linear programming relaxation of problem (1.3)–(1.9). A possible way to partially overcome this drawback is to consider only a limited subset of these constraints and to add the remaining ones only if needed, by using appropriate *separation procedures*. The considered constraints can be relaxed in a Lagrangian fashion, as done by Fisher [18] and Miller [39] (see Chapter 2), or they can be explicitly included in the linear programming relaxation, as done in branch-and-cut approaches (see Chapter 3). Alternatively, a family of constraints equivalent to (1.8) and (1.12) and having a polynomial cardinality may be obtained by considering the subtour elimination constraints proposed for the TSP by Miller, Tucker, and Zemlin in [38] and extending them to ACVRP (see, e.g., Christofides, Mingozzi, and Toth [7] and Desrochers and Laporte [12]):

$$u_i - u_j + Cx_{ij} \leq C - d_j \qquad \forall\, i, j \in V \setminus \{0\}, i \neq j, \text{ such that } d_i + d_j \leq C, \tag{1.13}$$

$$d_i \leq u_i \leq C \qquad \forall\, i \in V \setminus \{0\}, \tag{1.14}$$

where u_i, $i \in V \setminus \{0\}$, is an additional continuous variable representing the load of the vehicle after visiting customer i. It is easy to see that constraints (1.13)–(1.14) impose both the capacity and the connectivity requirements of ACVRP. Indeed, when $x_{ij} = 0$, constraint (1.13) is not binding since $u_i \leq C$ and $u_j \geq d_j$, whereas when $x_{ij} = 1$, they impose that $u_j \geq u_i + d_j$. (Note that isolated subtours are eliminated as well.)