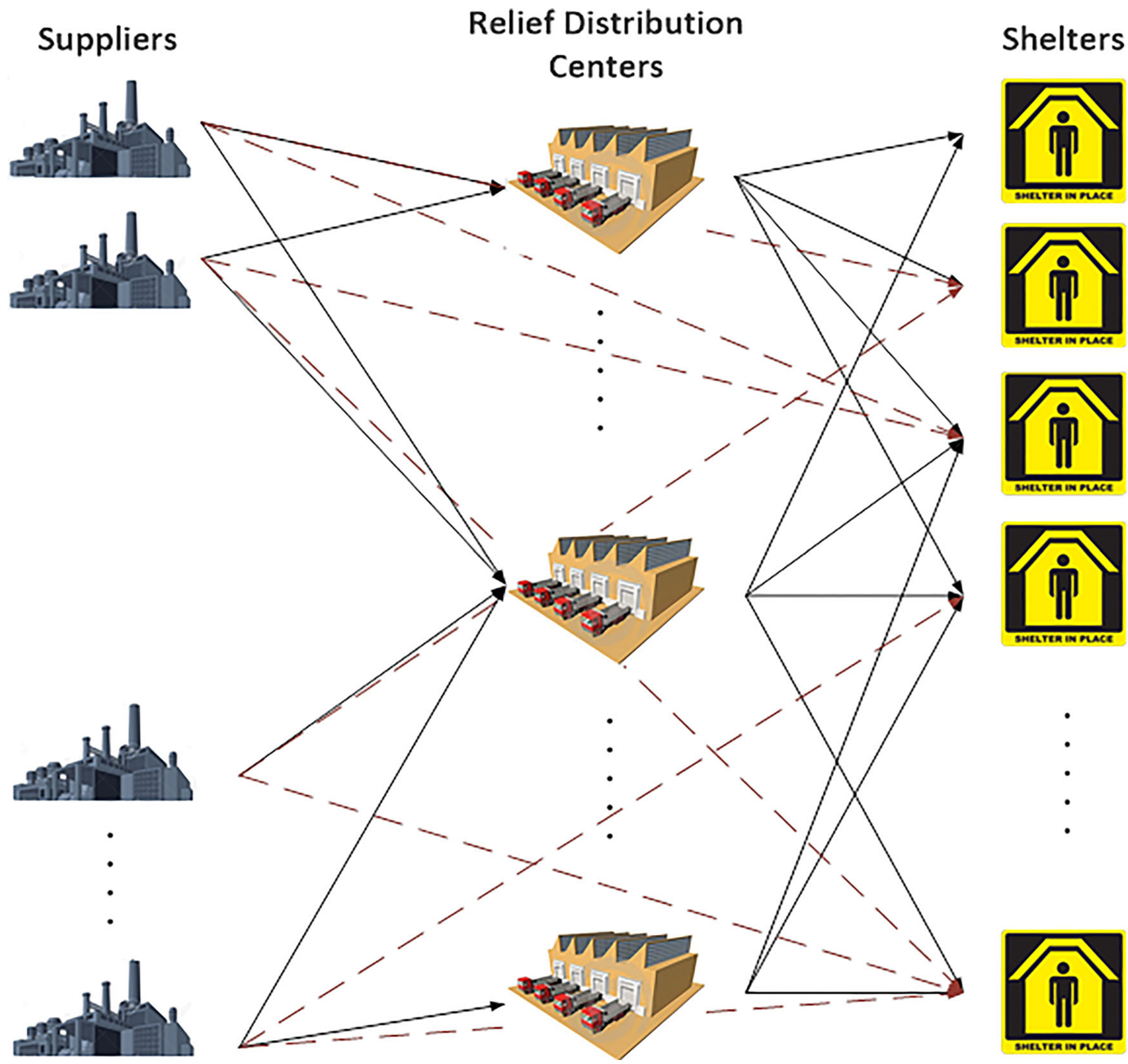

Figure 1. Schematic of the proposed network problem.

4. Problem model and formulation

In this section, a multi-objective mixed-integer stochastic programing model is formulated to obtain a sustainable humanitarian relief logistics network. The model aims to find the minimum total logistics cost; the maximum of the minimum rate of satisfaction level and the minimum total environmental impacts of the proposed humanitarian supply chain. The list of all indices, parameters, and variables used in the model formulation, are presented in Appendix C (Supplementary material).

4.1 Model formulation

The mathematical programing model developed for the SHRL problem is presented below.

4.1.1 Objective functions

The proposed problem has three objective functions indicated by Equations (1–3).

$$
\begin{aligned}
\min Z_1 = {} & \sum_{j\in J}\sum_{k\in K} F_{jk}.X_{jk} + \sum_{j\in J}\sum_{c\in C} h_{jc}.I_{jc} + \sum_{j\in J}\sum_{l\in L}\sum_{c_1\in C_1} b_{lc_1}.Q_{jlc_1} \\
& + \sum_{j\in J}\sum_{l\in L}\sum_{c_2\in C_2} b_{lc_2}.Q_{jlc_2} + \sum_{j\in J}\sum_{l\in L}\sum_{c\in C} \varphi_{jlc}.Q_{jlc} \\
& + \sum_{s\in S} p_s \Bigg[\sum_{j\in J}\sum_{c\in C} h_{jc}\Big(I_{jcs} - \sum_{i\in I} y_{ijcs} \Big) \\
& + \sum_{j\in J}\sum_{l\in L}\sum_{c_1\in C_1} b_{lc_1 s}.Q_{jlc_1 s} + \sum_{j\in J}\sum_{l\in L}\sum_{c_2\in C_2} b_{lc_2 s}.Q_{jlc_2 s} \\
& + \sum_{j\in J}\sum_{l\in L}\sum_{c\in C} \varphi_{jlcs}.Q_{jlcs} + \sum_{i\in I}\sum_{j\in J}\sum_{c\in C} \varphi_{ijcs}.y_{ijcs} \\
& + \sum_{i\in I}\sum_{l\in L}\sum_{c\in C} \varphi_{ilcs}.y_{ilcs} \Bigg]
\end{aligned}
\tag{1}
$$

$$
Max\ Z_2 = \sum_{s\in S} p_s \left(\min_i \left\{ \frac{\sum_{c\in C}\alpha_{ics}.\beta_{ics}}{\sum_{c\in C}\beta_{ics}} \right\} \right) \tag{2}
$$

$$\min Z_3 = \sum_{j\in J}\sum_{l\in L}\sum_{c_1\in C_1}\hat{b}_{lc_1}.Q_{jlc_1} + \sum_{j\in J}\sum_{l\in L}\sum_{c_2\in C_2}\hat{b}_{lc_2}.Q_{jlc_2} + \sum_{j\in J}\sum_{l\in L}\sum_{c\in C}\hat{\varphi}_{jlc}.Q_{jlc} + \sum_{s\in S}p_s\Bigg[\sum_{j\in J}\sum_{l\in L}\sum_{c_1\in C_1}\hat{b}_{lc_1}.Q_{jlc_1s} + \sum_{j\in J}\sum_{l\in L}\sum_{c_2\in C_2}\hat{b}_{lc_2}.Q_{jlc_2s} + \sum_{j\in J}\sum_{l\in L}\sum_{c\in C}\hat{\varphi}_{jlc}.Q_{jlcs} + \sum_{i\in I}\sum_{j\in J}\sum_{c\in C}\hat{\varphi}_{ijcs}.y_{ijcs} + \sum_{i\in I}\sum_{l\in L}\sum_{c\in C}\hat{\varphi}_{ilcs}.y_{ilcs}\Bigg] \quad (3)$$

4.1.1.1 Minimization of total logistics costs. The first objective function (Equation (1)) minimizes all logistics costs in the assumed humanitarian relief supply chain and includes two terms. The first term is about the first-stage costs for the preparedness phase consisting of opening costs of RDCs, inventory holding costs of relief items, procurement costs of different kinds of relief commodities, and transportation costs of relief commodities from suppliers to RDCs. The second term is related to the second-stage costs for the response phase including inventory holding costs, procurement costs, transportation costs from suppliers to RDCs, and transportation costs from suppliers and RDCs to the temporary shelters.

4.1.1.2 Maximization of the minimum satisfaction rate. The second objective function (Equation (2)) maximizes the minimum of satisfaction rates among the disaster areas in order to achieve fairness and social welfare purposes.

4.1.1.3 Minimization of the negative environmental effects. And finally, the third objective function (Equation (3)) minimizes the total undesirable environmental effects including ecological impacts of the packaging material of the relief goods, and CO_2 emissions in different transportation routes of the proposed network in both pre- and post-disaster phases.

The three mentioned objectives express three aspects of sustainability in a sustainable relief chain namely economic, social, and environmental aspects, respectively.

4.1.2. Problem constraints

4.1.2.1. Order quantity balance constraints.

$$Q_{jlc} = Q_{jlc_1} + Q_{jlc_2} \quad \forall j \in J,\ l \in L,\ c \in C,\ c_1 \in C_1,\ c_2 \in C_2 \quad (4)$$

$$Q_{jlcs} = Q_{jlc_1s} + Q_{jlc_2s} \quad \forall j \in J,\ l \in L,\ c \in C,\ c_1 \in C_1,\ c_2 \in C_2,\ s \in S \quad (5)$$

Constraints (4) and (5) denote that the total amount of a relief commodity procured from each supplier by each RDC is equal to the sum of the quantities ordered from different types of that commodity from that supplier in the pre- and postdisaster phases.

4.1.2.2 Inventory balance constraints.

$$I_{jc} = \sum_{l\in L} Q_{jlc} \quad \forall j \in J,\ c \in C \quad (6)$$

$$I_{jcs} = \sum_{l\in L} Q_{jlcs} + \lambda_{jc_1s}.\sum_{l\in L} Q_{jlc_1} + \lambda_{jc_2s}.\sum_{l\in L} Q_{jlc_2} \quad \forall j \in J,\ c \in C,\ c_1 \in C_1,\ c_2 \in C_2,\ s \in S \quad (7)$$

Constraint (6) states that the inventory level of each relief good in each RDC before the disaster is equal to the sum of quantities of that good purchased from different suppliers in the pre-disaster phase. Also, constraint (7) states that the inventory level of each relief good in each RDC after the occurrence of disaster is equal to the sum of quantities of that good purchased from different suppliers in the post-disaster phase plus the stored quantities of that item which remain usable.

4.1.2.3 Inventory level constraint.

$$\sum_{i\in I} y_{ijcs} \le I_{jcs} \quad \forall j \in J,\ c \in C,\ s \in S \quad (8)$$

Constraint (8) indicates that the total amount of relief items transferred from each RDC must be less than the inventory level of items in that specific RDC in the response phase.

4.1.2.4 RDC capacity constraints.

$$\sum_{c\in C} \theta_c.I_{jc} \le \sum_{k\in K} v_k.X_{jk} \quad \forall j \in J \quad (9)$$

$$\sum_{c\in C} \theta_c.I_{jcs} \le \sum_{k\in K} v_k.X_{jk} \quad \forall j \in J,\ s \in S \quad (10)$$

Constraints (9) and (10) represent the capacity limitations of RDCs. Also, constraints (8) to (10) ensure that an RDC could transfer relief goods to temporary shelters if it is opened already.

4.1.2.5 RDC location constraint.

$$\sum_{k\in K} X_{jk} \le 1 \quad \forall j \in J \quad (11)$$

Constraint (11) prevents placing more than one RDC at any possible location.

4.1.2.6 Pre- and Postdisaster supplier's capacity constraints.

$$\sum_{j\in J} Q_{jlc_1} \le v_{lc_1} \quad \forall l \in L,\ c_1 \in C_1 \quad (12)$$

$$\sum_{j\in J} Q_{jlc_2} \le v_{lc_2} \quad \forall l \in L,\ c_2 \in C_2 \quad (13)$$

$$\sum_{j\in J} Q_{jlc_1s} + \sum_{i\in I} y_{ilc_1s} \le v_{lc_1s} \quad \forall l \in L,\ c_1 \in C_1,\ s \in S \quad (14)$$

$$\sum_{j\in J} Q_{jlc_2s} + \sum_{i\in I} y_{ilc_2s} \le v_{lc_2s} \quad \forall l \in L,\ c_2 \in C_2,\ s \in S \quad (15)$$

Constraints (12) to (15) are the supplier's capacity limitations and guarantee that the quantities of different relief items purchased and transferred from no supplier can exceed the supplier's capacity for that specific item in both pre- and postdisaster phases.

4.1.2.7 Balance constraints of items shipped from suppliers.

$$y_{ilcs} = y_{ilc_1s} + y_{ilc_2s} \quad \forall i \in I,\ l \in L,\ c \in C,\ c_1 \in C_1,\ c_2 \in C_2,\ s \in S \tag{16}$$

Constraint (16) denotes that the total amount of each relief item transferred from one supplier to one Temporary Shelter (TS) is equal to the sum of that specific item with different kinds of packaging transferred from that supplier to that TS.

4.1.2.8 Postdisaster demand management constraints.

$$\alpha_{ics} = \frac{\sum_j y_{ijcs} + \sum_l y_{ilcs}}{d_{ics}} \quad \forall i \in I,\ c \in C,\ s \in S \tag{17}$$

$$0 \le \alpha_{ics} \le 1 \quad \forall i \in I,\ c \in C,\ s \in S \tag{18}$$

Constraints (17) and (18) represent the calculation of satisfaction rates of each TS for each relief item under each earthquake scenario, and as well the allowable range of these rates.

4.1.2.9 Non-negativity constraints.

$$X_{jk} \in \{0, 1\} \quad \forall j \in J,\ k \in K \tag{19}$$

$$I_{jc},\ I_{jcs},\ y_{ijcs},\ y_{ilcs},\ Q_{jlc_1},\ Q_{jlc_2}, Q_{jlc_1s},\ Q_{jlc_2s},\ Q_{jlc},\ Q_{jlcs} \in Z^+ \quad \forall i \in I,\ j \in J,\ l \in L,\ c \in C,\ c_1 \in C_1,\ c_2 \in C_2,\ s \in S \tag{20}$$

Finally, Constraints (19) and (20) are represented to determine the type of decision variables.

Linearization of the second objective function:

As the second function is non-linear, we linearize it using the auxiliary variable α_s and considering $\alpha_s = min_i \{\sum_{c\in C} \alpha_{ics}.\beta_{ics} / \sum_{c \in C} \beta_{ics}\}$;

$$Max\ Z_2 = \sum_{s \in S} p_s.\alpha_s \tag{21}$$

$$\alpha_s \le \frac{\sum_{c\in C} \alpha_{ics}.\beta_{ics}}{\sum_{c\in C} \beta_{ics}} \quad \forall i \in I,\ s \in S \tag{22}$$

$$\alpha_s \ge 0 \quad \forall s \in S \tag{23}$$

As a result, the final linear model of the assumed problem will be obtained by replacing the relation (21) by relation (2) and adding the relations (22) and (23) to other constraints of the proposed model.

5. Solution procedure

The mathematical model of our proposed problem is a multi-objective mixed-integer linear programing (MILP) model. The computational complexity of the MILP problem is NP-hard. On the other hand, solving the multi-objective optimization problems by different solution methods does not lead to the optimal values of all different objectives, so in these cases, it is better to use the multi-objective solution methods to solve the model. Several multi-objective solution methods have been applied to solve the multi-objective optimization models in the literature.

Since the proposed sustainability objective functions have different degrees of importance, based on the characteristics and advantages of Compromise Programing (CP) and Lexicographic Optimization (LO) methods, as well as the validation of the performance and effectiveness of these methods in solving similar multi-objective problems in the related literature such as Bozorgi-Amiri et al. (2013), Liberatore et al. (2014), Liu and Guo (2014), Ferrer et al. (2018), and Laguna-Salvadó et al. (2019), we use the CP and LO methods for our multi-objective SHRLP.

Therefore, in order to solve the proposed model handling the complexities of MILP and multi-objective optimization problem, we utilize the CP and LO solution methods then, we solve the transformed model using CPLEX solver of GAMS optimization software which uses the cutting plane method to solve the MILP problem and to find the optimal solutions.

The primary goal of humanitarian supply chain networks is to maximize supplying of demand of the affected people and minimize the effects of human suffering, so the social objective function (timely provision of relief items and justice in serving of the affected people) has the highest priority. Minimizing the environmental impacts of the relief logistics network is of secondary importance. And finally, although minimizing the cost of the relief logistics network is important, but since according to the approved finance laws, the provision of all costs (including the costs of establishing and opening distribution centers, transportation and distribution costs, and the procurement costs of relief items) is the responsibility of the National Committee for the Reduction of Disaster Effects in the Ministry of Interior, and as a result, these costs will be provided by the government and the Municipality of Tehran, as well as some of these costs would be provided by the donations of humanitarian and non-government organizations. Therefore, the degree of importance of the cost objective function is considered less than two other objectives.

5.1. Compromise programing technique

Applying the Compromise Programing (CP) method, a non-dominating compromise solution could be obtained for a multi-objective problem by minimizing the summation of the normalized differences between optimal values of objective functions and the respective objective functions.

Based on the Lp-metrics method, the solution procedure steps for our proposed problem are as follows:

Step 1: Solve the problem considering only one objective function Z_i $(i = 1,\ 2,\ 3)$ for all of the objectives separately and without regarding all the other objectives.

Step 2: Determine the optimum value of each objective function from step 1 as; Z_1^{min}, Z_2^{max}, Z_3^{min} respectively.

Step 3: Reformulating the following single objective problem considering Z_4 as the Lp-metrics objective function;