The sets I and J denote the sets of arriving and departing train units, respectively. The arriving and departing train units have been ordered according to their certain arrival and departing time. Each train unit has a certain type p_i and p_j, depending on whether the train unit departing or arriving one. an arriving train unit can be only matched with a departing one if and only if they have the same type. According to these types, for each arriving i_train unit (iϵI) there is a set J_i (J_i⊆J) of departing train units that can be matched with arriving i_train unit. It is same vice verses for each departing j_train unit with a set I_j (I_j⊆I). The length of each train unit is denoted by WL. Also let, T denote the set of the shunt tracks. The length of shunt track tϵT is donated ly L(t). The shunting cost to or from shunting track t is denoted by c(t).

The model contains binary variables Z_{it} equal to 1 if and only if arriving i_train unit is parked at shunt track t and Z_{jt} equal to 1 if and only if departing j_train unit is parked at shunt track t. Additionally, binary variables x_{ijt} equal to 1 if and only if arriving i_train unit is matched with departing j_train unit at shunt track t. Model is given as flows:

$$\min \sum_{t\epsilon \mathrm{T}} c_t \left(\sum_{i\epsilon \mathrm{I}} z_{it} + \sum_{j\epsilon \mathrm{J}} z_{jt} \right) \tag{1}$$

$$\sum_{t\epsilon \mathrm{T}} z_{it} = 1, \qquad i \ \epsilon \ I , \tag{2}$$

$$\sum_{t\epsilon \mathrm{T}} z_{jt} = 1, \qquad j \ \epsilon \ J , \tag{3}$$

$$z_{it} = \sum_{j\epsilon J_i} X_{ijt,} \qquad t \ \epsilon \ T , \quad i \ \epsilon \ I , \tag{4}$$

$$z_{jt} = \sum_{i\epsilon I_j} X_{ijt,} \qquad t \ \epsilon \ T , \quad j \ \epsilon \ J , \tag{5}$$

$$\sum_{j'\epsilon \mathrm{J_i}:j'>j} X_{ij't} + \sum_{i'\epsilon I_j:i'<i} X_{i'jt} \leq 1,$$

$$t \in T, i \in I, j \in J, i < j, \tau_i \neq \tau_j, \tag{6}$$

$$\sum_{i' \in I: i' \leq i} l_{i'} * z_{i't} - \sum_{j' \in J: j' < i} l_{j'} * z_{j't} \leq l_t,$$

$$t \in T, i \in I, \tag{7}$$

$$z_{it} \in \{0,1\}, \quad i \in I, \quad t \in T, \tag{8}$$

$$z_{jt} \in \{0,1\}, \quad j \in J, \quad t \in T, \tag{9}$$

$$x_{ijt} \in \{0,1\}, \quad i \in I, \quad j \in J, \quad t \in T, \tag{10}$$

With constraint in equation 6, I am trying to handle the restriction for crossing which is one of the special constraint in shunting. Crossing means a train unit obstructing another train unit during its departure or arrival at a shunt track.

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eq6(i,j,t)$(ord(i) < ord(j)).. sum(j, X(i,ord(j)-'1',t)) + sum(i, X(ord(i)-'1',j,t)) =l= 1;
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Above constraint is the crossing constraint and ensure that on a single shunting track, the arriving and departing train units are matched in a LIFO (last in first out) way. For much more clarification, this constraint make certain that if there are other train units than departing j_train unit at track t at the moment that train unit j wants to depart from shunting track t, then these train units have arrived earlier at shunting track t than departing j_train unit. Consequently, departing j_train unit is the first train unit at shunting track t and can depart from shunting tack t without any crossing.

Considering that, this constraint is only relevant for (p_ i ≠ p_ j). Actually, (p_ i = p_ j) refers that

(p_j-1=p_i=p_j=p_i+1) and if all these train units have the same type then any crossing can be occurred. Because same train units can be exchanged if necessary.